

7 Prerequisites:-

The term data structure is used to describe the way the data is stored  
The algorithm is used to describe the way it is processed  
So Algorithm and data structures are interrelated

choosing a different data structure affects the algorithm we might use and algorithm chosen affects the data structure we choose

Algorithm → The method of solving a problem is called Algorithm

It is a sequence of instructions that act on input data to produce some output in a finite number of steps is called an algorithm

An Algorithm must have following Properties

Input

An algorithm requires an input supplied externally

Output

It must produce at least one output

finiteness

no matter what it must terminate at some point

definiteness

It must be clear and unambiguous and should be understood by a kindergarten student

effectiveness

One must be able to perform the steps without any intelligence

**\*\*** Algorithm + data structure = program

Data structure

It is a logical relationship existing between individual elements of data

ADT (Abstract data type)

It is a mathematical model or concept or that defines a data type logically

data structure is a programming construct used to implement an ADT

- List ADT
- stack ADT
- queue ADT

Three ADT's are differentiated by types of operations performed on them

It is the physical or actual representation of ADT

ADT Datastructure

ADT tells us what is to be done and data structure tells us what and how it is to be done

A program that generally uses a data structure is called as a client. The specification of ADT is called the interface and it is the only thing that is visible to the client programs that uses the data structure

Advantages of datastructure

efficiency

Proper choice of data structures makes our program efficient

Reusability

data structures are reusable; once they are implemented they can be used at any other place

Abstraction

The client program uses the data structure through only interface only without getting into the implementation details

Linear data structure & non linear data structure

If all the elements are arranged in sequential order then it is called linear data structure

If there is no linear order arrangement of elements then it is called non linear data structure

In linear data structure each element has a only one predecessor and successor

except the last and first elements as first element has no predecessor and last element has no successor

array, linked list, string, stack and queue are examples

trees and graphs are examples

Based on memory there are two types of data structures

static data structure

memory is allocated at compilation and maximum size is fixed

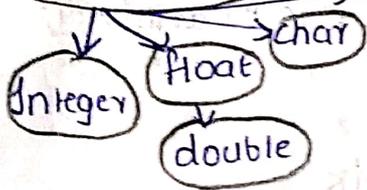
dynamic datastructure

memory is allocated at run time and maximum size can be changed at run time

# Basic classification of data structures

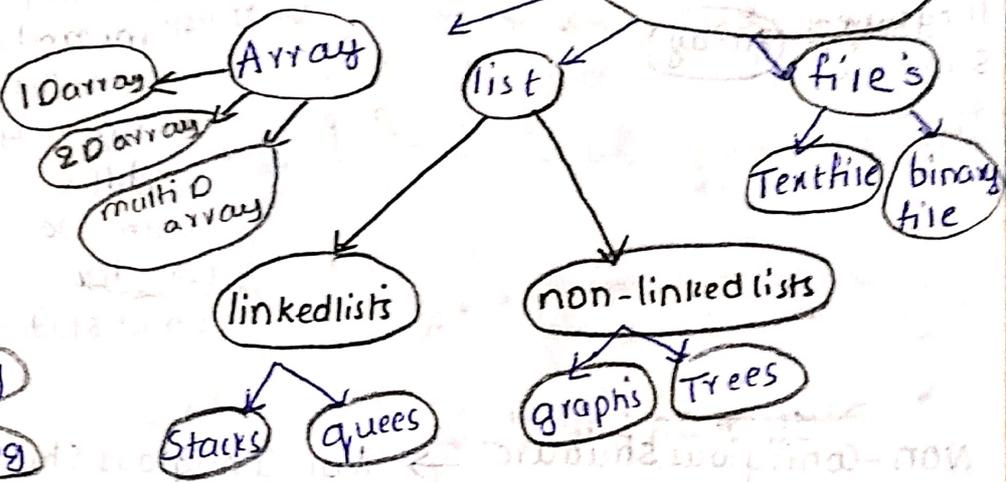
Which are basic types and runs on the machine instructions

Primitive data structures

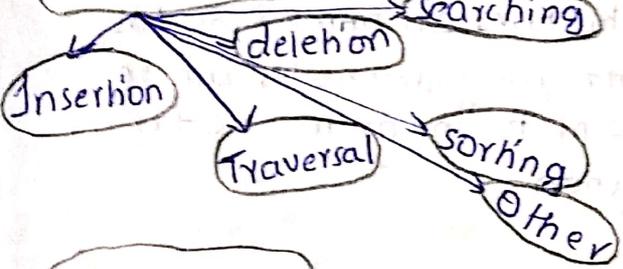


Which are complicated and derived from the primitive data structures

non primitive data structures



Operations performed on data structures



## Organization of data

Contiguous

non contiguous

hybrid

In contiguous structure the elements are kept together in memory ex: Array

The basic difference of contiguous and non contiguous are can be based on the terms of data stored in the memory

The collection of data you work with program have some kind of organization or structure

even though how complex our data structures can be they can be splitted into three types

In non contiguous structure the elements are scattered in the memory but we link them in some way eg: linked lists

here the nodes of the list are linked together using pointers stored in each node

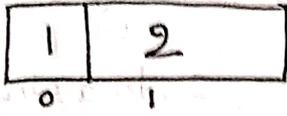
# Contiguous structure

They may be simply classified into two types

Contiguous Structures with same size

As each element is of same type they have same size

```
int array[3] = {1, 2}
```

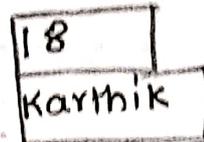


Array

Contiguous structures with different size

```
struct student  
{  
    int age;  
    char name[20];  
};  
student s = {18, "Karthik"};
```

As it contains int and char the size may differ when we consider



Structure

struct student as a data structure

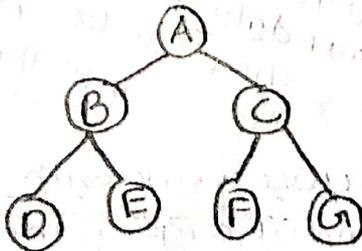
# Non-Contiguous structure

Non contiguous structures are implemented as a collection of data items called nodes, where each node can point to one or more other nodes in the collection

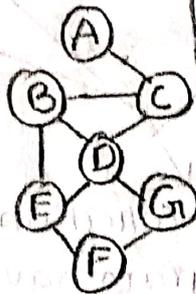
Examples:



(a) Linked list



(b) Tree



(c) graph

A linked list is a one dimensional non contiguous structure

A tree is a two dimensional non contiguous structure

Whereas the graph has no restrictions like tree as to non up, down, left right

### Hybrid structure

If two basic types of structures are mixed then it is called a hybrid structure. Then one part is contiguous and another part is non contiguous.

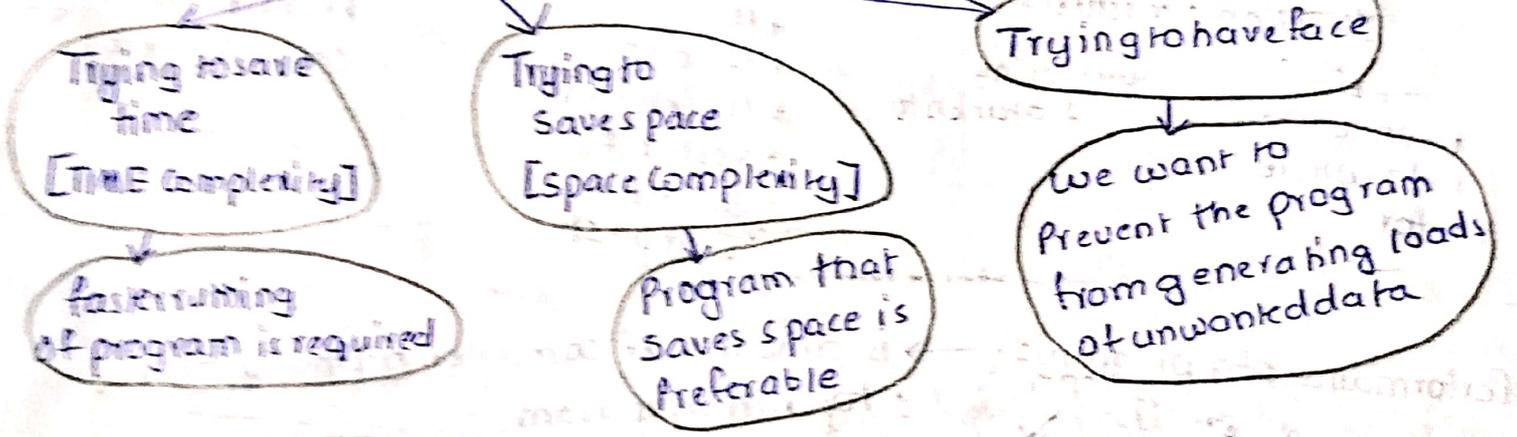
### Conceptual structure



The implementation of a double linked list using three parallel arrays and stored a part from each other in memory.

### Problems faced when designing a practical Algorithm

These are also the basic design goals we must strive in a program



### Approaches to design an algorithm

#### Top-down approach

Generally structured oriented programs use Top-down approach

It is also called as step-wise approach

In this approach the programmer needs to write code for the main-function. In main function they will call other functions

It requires the good understanding of program

#### bottom-up approach

Generally Object oriented programming languages use this approach

In this approach the programmer has to write code for the modules. Then they have to look for the integration of modules

It is suitable for the projects that can be developed from existing projects

Note: Topdown approach first focuses on abstract of overall system or project. At last it focuses on detail design or development

It goes from high level design to low level design (or) system (TOP-down approach)

Bottom-up approach first focuses on detail design or development. At last it concentrates on the abstract of overall system or design

Nowadays modern software designs combine both these approaches for the better results

Bottom up approach starts from low level design to high level design or system

Control structures used in Algorithm statements

Sequence statements

Selection structures

Branching statements

Iteration structures

or looping statements

- \* while
- \* do while
- \* for

- \* if
- \* if-else
- \* if-else-if
- \* nested if
- \* switch
- \* Jump-st

PRINT, READ, CALCULATE etc

Performance of a program:

is amount of computer memory, time to run a program

we use two approaches to determine the performance of a program

due to the platform dependence and machines, the results of the logical approach may vary with actual figures

Analytical (or) logical approach

In analysis we use this method

In which we approximately calculate the memory, space using the logical approach

Experimental approach

But while measurement we use this approach

In which we calculate exact amount of time, memory accurately

These are generally actual figures

# Time Complexity

Time needed by algorithm expressed as a function of size of problem is called time complexity of an algorithm

It is the time required for the completion of program

The limiting behaviour of the complexity as size increases is called asymptotic time complexity

Asymptotic time complexity ultimately determines the size of problems that can be solved by algorithm

# Space Complexity

is the amount of memory required by a program for its completion

Components of program occupying space

Environment stack space

Dataspace

It is the space needed to store all constant and variable values

Instruction space

This space required depends on some factors

is used to save information needed to resume execution of partially completed functions

The compiler used to complete the program into machine code

The compiler options in effect at the time of compilation

The target computer

Instruction space

It is the space needed to store the compiled version of program instructions

Space needed by constants and variables

Space needed by dynamically allocated objects such as arrays and class instances

Which may be dynamically allocated at the runtime at the convenience of the user

It is of two types

# Classification of the algorithm

If  $n$  is the number of data items to be processed or degree of polynomial or the size of list to be sorted or searched or nodes in a graph

1

Next instructions of small programs are executed at once or mostly a few times, then we say the running time is constant

$\log n$

The program gets slightly slower as  $n$  grows. This running time constantly occurs in the programs that solve a big problem by transforming into smaller problem. Whenever  $n$  doubles  $\log n$  increases by a constant, but  $\log n$  never doubles until  $n$  becomes  $n^2$

$n$

This occurs when running time is linear. In this case small amount of processing is done on each input element

$n \cdot \log n$

This running time occurs when we solve problem by breaking up into smaller problems and solve them independently. If  $n$  doubles, running time more than doubles

$n^2$

When the running time of algorithm is quadratic, it can be used practically on relative small problems. In case of double nested loop, if  $n$  increases running time increases four fold. But in case of  $n^3$  it increases 8 fold times. And these process (triples of data (eg: triple nested loop)).

$2^n$

These algorithms with exponential running time are appropriate for practical use. Such algorithms are brute-force solution to problems. If  $n$  doubles, running time squares

Complexity of algorithm

Complexity function  $f(n)$  which gives running time / storage of algorithm in terms of size  $n$  of input data.

The storage space required by algorithm is simply a multiple of data of size  $n$

Complexity shall refer to the running time of algorithm

The complexity of an algorithm also depends on the particular data

Best case

Average case

The expected or predicted value of  $f(n)$

Worst case

The minimum possible value of  $f(n)$  is called best case

The field of Computer science which studies efficiency of algorithms is called analysis of algorithms

The maximum value of  $f(n)$  for any  $f(n)$  for any key possible input

Rate of growth

These notations are used in performance analysis

Also use to categorize complexity of algorithm

**Big-O ( $O$ ) (Upper bound)**

$f(n) = O(g(n))$

(Pronounced as order of  $(n)$ )

big  $O$  says that growth rate of  $f(n)$  is less than or equal ( $\leq$ ) that of  $g(n)$ ;  $n \geq n_0$ ;  $c > 0$ ;  $n_0 \geq 1$

**Big omega ( $\Omega$ ) (Lower bound)**

$f(n) = \Omega(g(n))$

Says that growth rate of  $f(n)$  is greater than or equal to that of  $g(n)$

**Big-Theta ( $\Theta$ ) (Same) order)**

$f(n) = \Theta(g(n))$

Says that growth rate of  $f(n)$  equals to the growth rate of  $g(n)$ .

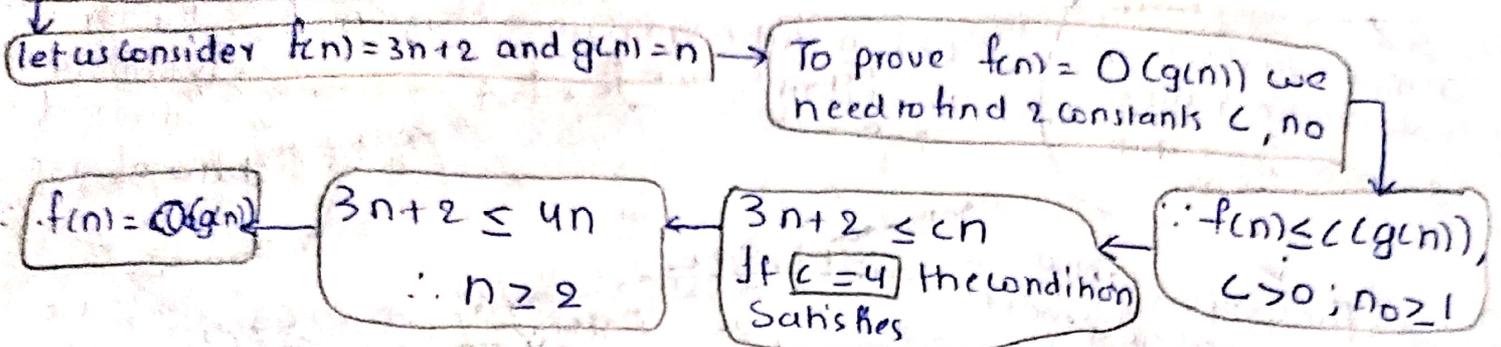
(If  $f(n) = O(g(n))$  and  $T(n) = \Omega(g(n))$ )

$c_1 g(n) \leq f(n) \leq c_2 g(n)$

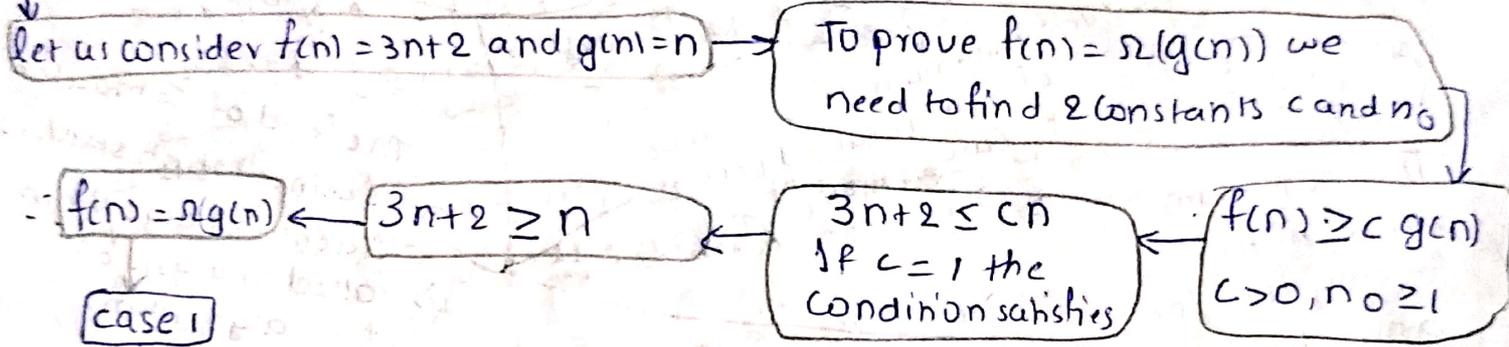
Says growth rate of  $T(n)$  is less than  $P(n)$  if  $T(n) = O(P(n))$  and  $T(n) \neq \Theta(P(n))$

Little- $o$  ( $o$ )  
 $T(n) = o(P(n))$

### Proof for Big Oh (O)



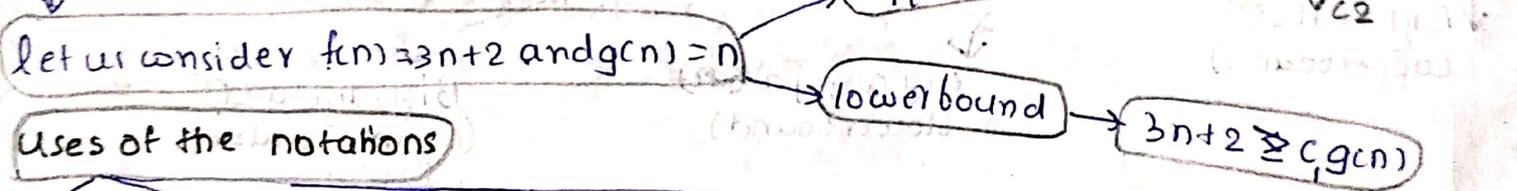
### Proof of Big Omega (Ω)



case 1

we cannot take any value lower bounded by  $n^2$  but we can take a value upper bounded by  $n^2$  and values less than  $n$  like  $\log n$

### Proof of Theta (Θ)



### Uses of the notations

#### Big Oh (O) (Upper bound)

It is used to represent the worst case of an algorithm. It is the maximum time taken and it does not exceed this

In practice we actually tend to find the best case rather than the best case

gives max time

#### Big Omega (Ω) (lower bound)

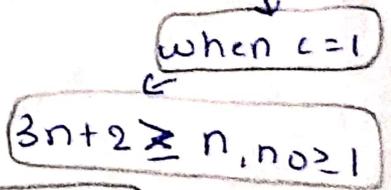
It is used to represent the best case of an algorithm. In any case we cannot achieve better than this

We use this to show the least possible time taken by algorithm for its execution

#### Big-Theta (Θ)

It is used to represent the average case of an algorithm

We use average case if both best case and worst case are same i.e. takes same time to any input



# Rules for O notation

## Transitivity

If  $f(n) = O(g(n))$   
and  $g(n) = O(h(n))$   
then  $O(h(n))$

If  $f_1(n)$  is  $O(h(n))$  and  $f_2(n)$  is  $O(h(n))$   
then  $f_1(n) + f_2(n)$  is  $O(h(n))$

If  $f_1(n)$  is  $O(h(n))$  and  $f_2(n)$  is  $O(g(n))$   
then  $f_1(n) + f_2(n)$  is  $\max(O(h(n)), O(g(n)))$

If  $f_1(n) = O(h(n))$  and  $f_2(n) = O(g(n))$   
then  $f_1(n)f_2(n) = O(h(n)g(n))$

If  $f(n) = c \Rightarrow f(n)$  is  $O(1)$

If  $f(n) = c * h(n)$

Any polynomial  $p(n)$  of degree  $m$  is  $O(n^m)$

$n^a$  is  $O(n^b)$  only if  $a \leq b$

All logarithms grow at same rate while computing O notation, base of the logarithm is not important

let  $x^a = \log_a n$  and  $y^b = \log_b n$   
 $a^{x^a} = n$  and  $b^{y^b} = n$   
 $a^{x^a} = b^{y^b}$   
Taking  $\log_a$  of both sides  
 $\log_a a^{x^a} = \log_a b^{y^b}$   
 $x^a * \log_a a = y^b * \log_a b$   
 $x^a = y^b * \log_a b$

Recursive procedures work well for many problems

Recursion

one of the complicated way of solving problems even though programmers prefer it even if simpler solutions are available

It costs more in terms of time and space

It is a programming technique used to solve problems in terms of similar problems

It is very hard to use recursion for huge problems

A recursive function must call itself recursively

Laws of Recursion

A recursive algorithm must have a base case

A recursive algorithm must change its state and move towards base case

Base case is also called the terminating case

If there is no base case, then it is called infinite recursive function

Modularization

It shows solving or splitting up large problem into smaller parts and solving it

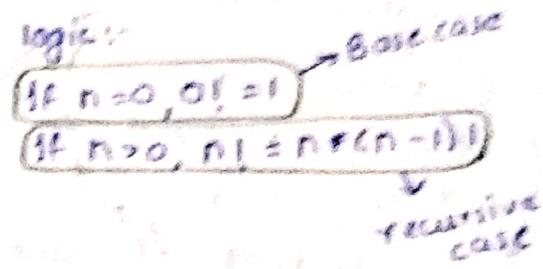
In searching and sorting techniques we use this procedure to split the big problem into parts and solving them and combining them to form the actual solution

Q\*\* Program to find the factorial of a number using recursion

```
#include <stdio.h>
int factorial (int n);
void main()
{
    int num, fact;
    printf ("Enter a positive integer value");
    scanf ("%d", &num);
    fact = factorial (num);
    printf ("n factorial = %d", fact);
}
int factorial (int n)
{
    int result;
    if (n == 0)
        return 1;
    else
        result = n * factorial (n-1);
    return result;
}
```

non-recursive function

```
factorial (int n),
{
    int i, result = 1;
    if (n == 0)
        return 1;
    else
    {
        for (i = 1; i <= n; i++)
            result = result * i;
    }
    return result;
}
```



let n = 5

$5! = 5 * 4!$       $fact(5) = 5 * fact(4)$   
 $4! = 4 * 3!$       $fact(4) = 4 * fact(3)$   
 $3! = 3 * 2!$       $fact(3) = 3 * fact(2)$   
 $2! = 2 * 1!$       $fact(2) = 2 * fact(1)$   
 $1! = 1 * 0!$       $fact(1) = 1 * fact(0)$   
 $0! = 1$               $fact(0) = 1$

$5! = 5 * 4!$   
 $= 5 * 4 * 3!$   
 $= 5 * 4 * 3 * 2!$   
 $= 5 * 4 * 3 * 2 * 1!$   
 $= 5 * 4 * 3 * 2 * 1 * 0!$   
 $= 120$

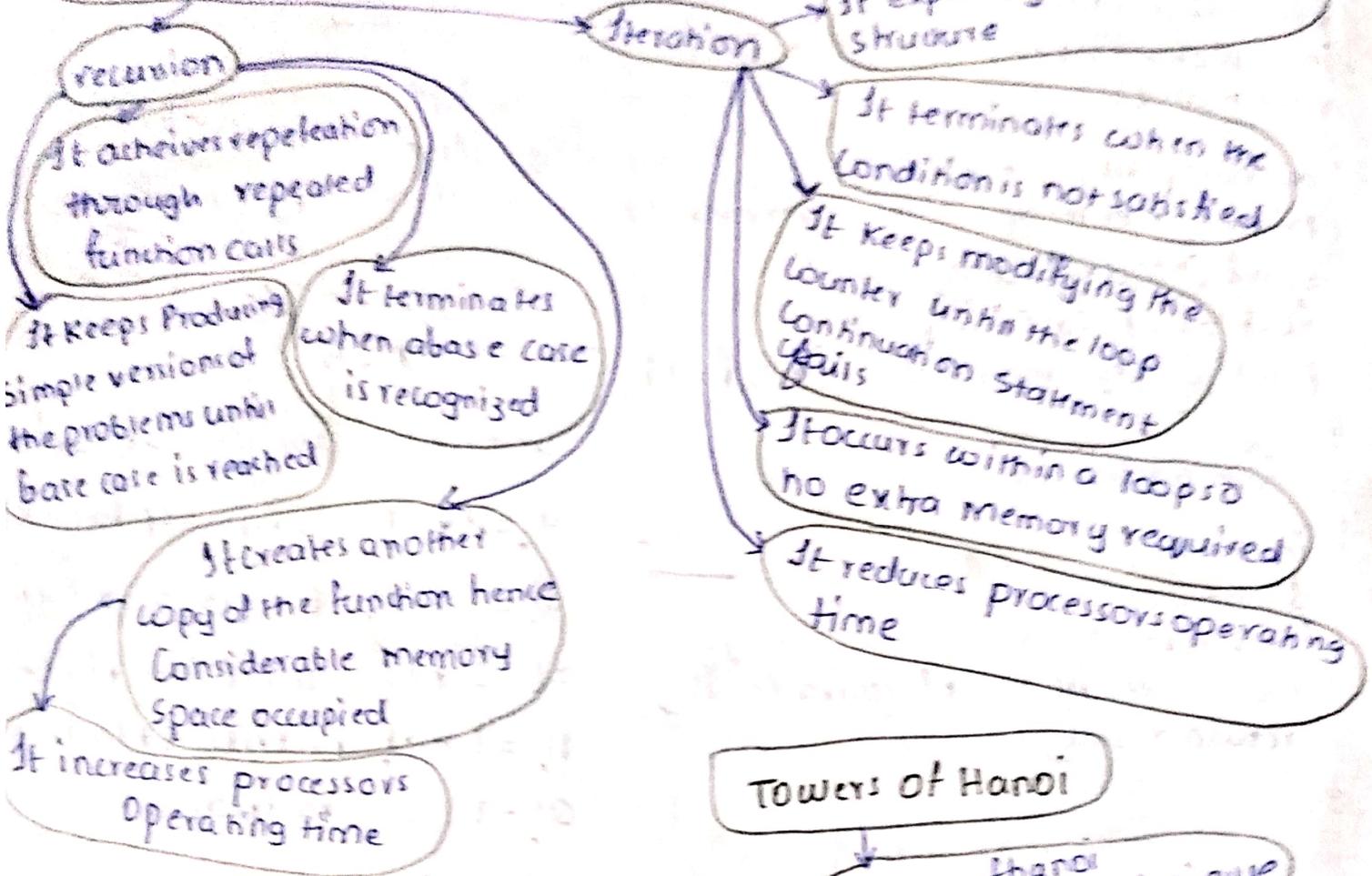
Similarities involved in both recursion and iteration

Both of the techniques involve repetition

Both of the solving methods have a terminating case or a condition

Both can occur infinitely if there is no terminating condition

# Differences between recursion and iteration



## Types of recursion

### Binary recursion

If the function repeats itself twice then it is called binary recursion

```

int yfiboc(int n)
{
    if(n == 1)
        return 0;
    else if(n == 2)
        return 1;
    else
        return yfiboc(n-1) + yfiboc(n-2);
}
  
```

### Tail recursion

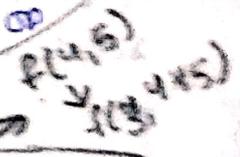
It is a form of linear recursion. If a recursive function is last case and there are no pending operations, on return recursive call, then that function is said to be tail recursion.

```

#include <stdio.h>
int tailf(int n, int r);
void main()
{
    int r = 1;
    r = tailf(n, r);
}
int tailf(int n, int r)
{
    if(n == 1)
        return r;
    else
        return tailf(n-1, n*r);
}
  
```

## Towers of Hanoi

In towers of hanoi we use recursion technique



# Designing and developing a recursive function

## STEP: 1

determine the Base case or recursive case / simple case:  
 > It is the terminating condition of a recursive function  
 > It never contains a function call

## STEP 2

determine the recursive case / general case in which the program is expressed in terms of similar problems

## STEP 3

Combine Base case and recursive case to complete recursive function

The recursive function never contains any loops like while, do while

## NOTE

## EXAMPLE logics

To print numbers from 1 to n using recursion

```

f(i, n) i = 1
if (i = n) print n
if i < n, print i
f(i+1, n)
    
```

To find  $x^y$  value using recursion

```

p(x, y)
if (y == 0)
    return 1
if (y > 0)
    x * p(x, y-1)
    
```

## Applications of data structure

Simulation and modeling software

Artificial intelligence

database management engine

Computer networks

gaming software

Compiler design

operating system

numerical analysis

## Towers of Hanoi

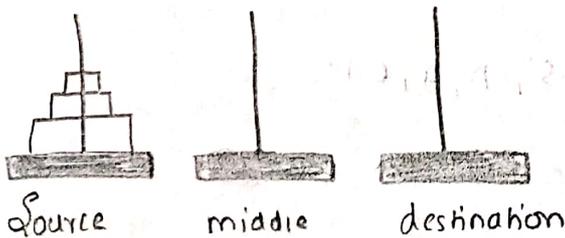
It is a classic recursion game.

## Rules

1. All disks must be of different size
2. Only one should disks should be moved at a time
3. only top most disk can be moved
4. A larger disk cannot be placed on a smaller disk

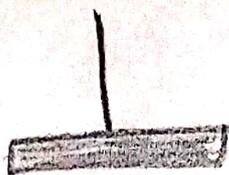
It contains 3 towers namely source, middle, destination

The target of the game is to move the disks from source tower to destination tower using middle tower



Initial position of disks in hanoi towers game

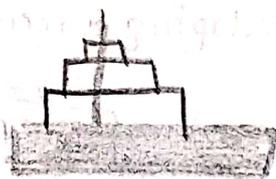
The towers of hanoi problem can be easily solved using recursion.



Source



middle



destination

Final setup of towers of hanoi.

// Program to solve towers of hanoi problem / game

```
#include <stdio.h>
```

```
void towers_of_hanoi(int n, char *a, char *b, char *c);
```

```
void main()
```

```
{ int n;
```

```
printf("Enter how many disks you want : ");
```

```
scanf("%d", &n);
```

```
towers_of_hanoi(n, "source", "auxiliary", "destination");
```

```
void towers_of_hanoi(int n, char *a, char *b, char *c)
```

```
{ if (n == 1)
```

```
{ printf("In %d: move disk 1 from %s to %s", n, a, c);
```

```
}
```

```
else
```

```
{ towers_of_hanoi(n-1, a, c, b);
```

```
printf("In %d move disk %d from %s to %s", n, n, a, c);
```

```
towers_of_hanoi(n-1, b, a, c);
```

```
}
```

\* If there are 'n' disks in the game then there will be  $2^n - 1$  of steps

**Fibonacci's sequence problem** → A fibonacci sequence starts with 0 and 1.

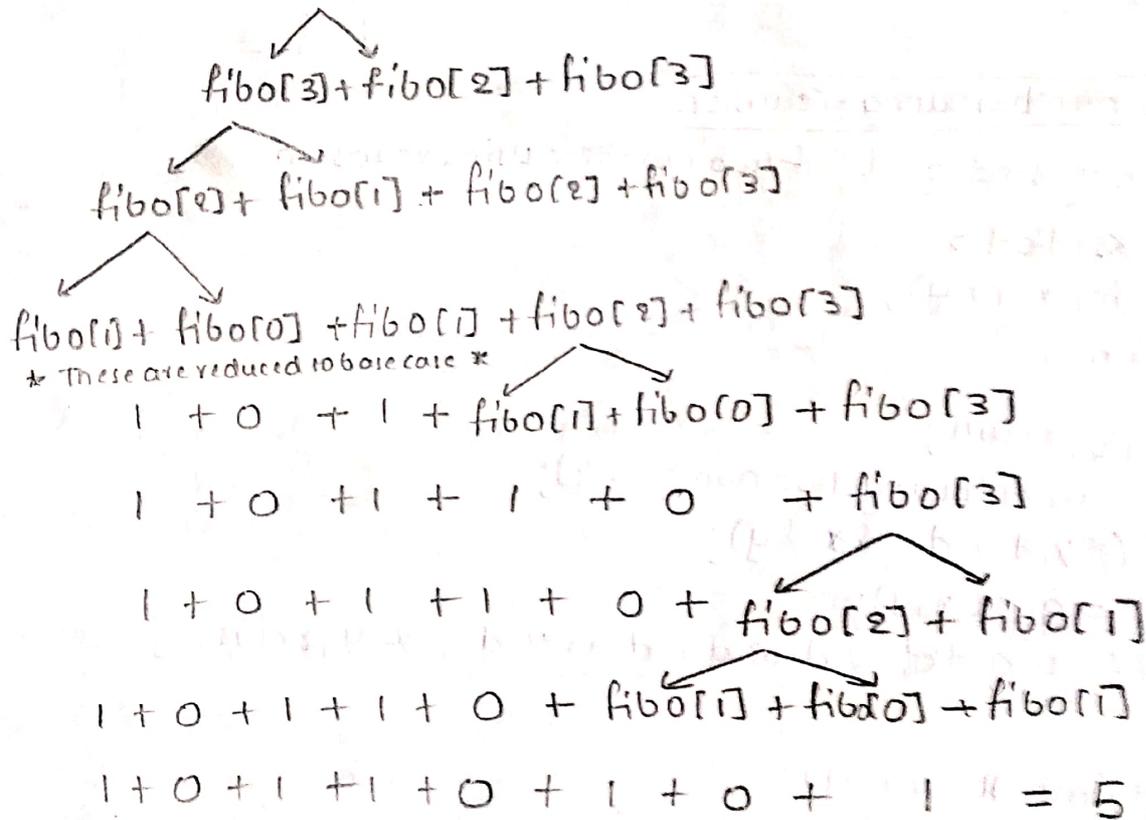
→ Successive elements are obtained by summing the preceding two elements in sequence

∴ 0 1 1 2 3 5 8 13 21

→ A recursive function can be used to find  $n^{\text{th}}$  number in fibonacci sequence.

$$\begin{aligned} \text{Fib}(n) &= n \text{ if } n=0 \text{ or } n=1 \\ \text{Fib}(n) &= \text{Fib}(n-1) + \text{Fib}(n-2) \text{ for } n \geq 2 \end{aligned}$$

eg:  $\text{fib}(5) = \text{fib}(4) + \text{fib}(3)$



\* fib(2) ↓ 3 times  
\* fib(3) ↓ 2 times

NOTE: we save the values of fib(2) and fib(3) and use them if needed later.

```
// Recursive function to compute the fibonacci number using nth position given
#include <stdio.h>
int fibo(int x);
void main()
{
    int n, result;
    printf("Enter the position of in fibonacci series you want: ");
    scanf("%d", &n);
}
```

```

1 result = fibo(n);
  printf("In the number in %d position is %d", n, result);
}
int fibo(int x)
{
  if((x == 0) || (x == 1))
    return x;
  else
    return fibo(x-1) + fibo(x-2);
}

```

### GCD of two numbers using recursion

// Program to find gcd of two numbers using recursion

```

#include <stdio.h>
int gcd(int x, int y);
void main()
{
  int x, y, result;
  printf("In entering two numbers ");
  scanf("%d %d", &x, &y);
  result = gcd(x, y);
  printf("In gcd of %d and %d is %d", x, y, result);
}
int gcd(int x, int y);
{
  while(x != y)
  {
    if(x > y)
      return gcd(x-y, y);
    else
      return gcd(x, y-x);
  }
  return x;
}

```

There are Basically two aspects of Computer programming

data organization

data structure

choosing right and appropriate algorithm

Problem solving

To learn this Problem solving we need to study searching and sorting of data

Searching

It allows the efficient arrangement of elements within the data structure

Sorting

classification of elements in ascending or descending order

It is used to find the location of the element if it is available

Internal Sorting methods

External sorting methods

They are basically two types

Insertion sort  
merge sort

Bubblesort

Quicksort

Shell sort

heapsort

Radix sort

Linear search (or) sequential search

Binary Search

Linear search:

- > It is the simplest of all searching techniques.
- > In this technique ordered or unordered list be searched with the target value one by one from beginning till the desired element is found.
- > If the desired element is found search is successful otherwise it is unsuccessful.

Suppose there are n elements organized on the list sequentially

The number of comparisons required to retrieve an element purely depends on the where element is stored

If the search is unsuccessful you need 'n' comparisons.

1<sup>st</sup> element = target (or) 2<sup>nd</sup> (or) 3<sup>rd</sup>

Advantages

- It is simple
- It works for Ordered or unordered list

Disadvantages

- It is only efficient when the number of elements are less

$\frac{(n+1)}{2}$  average comparisons u need

```

// Program to perform Linear search using non-recursion.
#include <stdio.h>
void linearsearch (int A[], int x); // correction **
void main ()
{
    int A[20], n, target, i;
    printf("In enter how many value to store in array");
    scanf("%d", &n);
    printf("In enter values into array");
    for(i=0; i<n; i++)
    {
        scanf("%d", &A[i]);
    }
    printf("In enter target value to be searched");
    scanf("%d", &target);
    linearsearch(A, target); // (A, target, n)
}

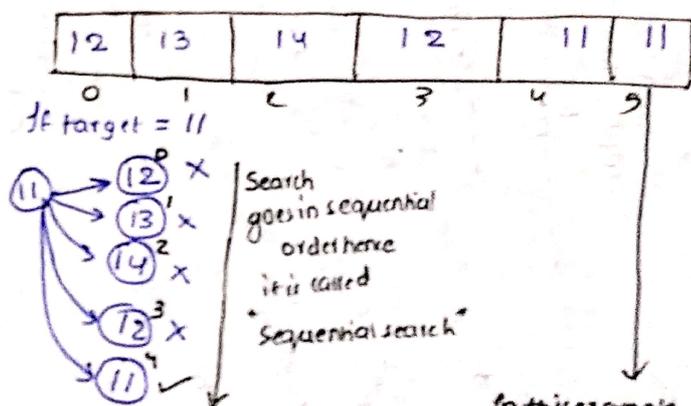
void linearsearch(int A[], int x) // (int A[], int x, int n)
{
    int index = 0, flag = 0;
    while (index < n)
    {
        if (x == A[index])
        {
            flag = 1;
            break;
        }
        index++;
    }
}

```

```

if (flag == 1)
    printf("Data found at %d position", index);
else
    printf("Data not found");
}

```



In this example the last element = target But we already found at 4<sup>th</sup> position we cannot find search also the repeated elements cannot be searched using this mechanism.

```

// Program to perform Linear search using recursion.
#include <stdio.h>
void linearsearch(int a[], int data, int position, int n);
void main ()
{
    int a[20], i, n, data;
    printf("In enter number of elements");
    scanf("%d", &n);
    printf("In enter elements");
    for(i=0; i<n; i++)
    {
        scanf("%d", &a[i]);
    }
    printf("In enter element to be searched");
    scanf("%d", &data);
    linearsearch(a, data, 0, n);
}

void linearsearch(int a[], int data, int position, int n)
{
    if (position < n)
    {
        if (a[position] == data)
            printf("In data found at %d", position);
        else
            linearsearch(a, data, position+1, n);
    }
    else
        printf("In data not found");
}

```

